

Dynamical origin and the pole structure of $X(3872)$

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The dynamical mechanism of channel coupling with the decay channels is applied to the case of coupled charmonium - DD^* states with $J^{PC} = 1^{++}$. A pole analysis is done and the DD^* production cross section is calculated in qualitative agreement with experiment. The sharp peak at the $D_0D_0^*$ threshold and flat background are shown to be due to Breit-Wigner resonance, shifted by channel coupling from the original position of 3954 MeV for the 2^3P_1 , $Q\bar{Q}$ state. A similar analysis, applied to the $n = 2$, 3P_2 , 1P_1 , 3P_0 , allows us to associate the first one with the observed $Z(3930)$ $J = 2$ and explains the destiny of 3P_0 .

The resonance $X(3872)$ found in [1] and confirmed and further studied in [2–5] (see [6] for review) is still a mysterious phenomenon. The measured quantum numbers of $X(3872)$ [2] suggest that this is a 1^{++} state. One can list several properties of this resonance which are difficult to explain. (1) The width of the peak at 3872 MeV is zero within experimental energy resolution. (2) The peak is exactly at the $D_0D_0^*$ threshold (3871.2 MeV) and not at a little higher $D_+D_-^*$ threshold (3879 MeV); however, isospin conservation predicts that both thresholds should enter with the same weight. (3) The single-channel theory [7] predicts a standard 2^3P_1 level of the $Q\bar{Q}$ system around 3950 MeV; however, among the structures observed by Belle in this region, $X(3940)$, $Y(3940)$, $Z(3930)$, there seems to be no examples suggesting the 1^{++} identification [6]. (4) Among the four members of the $2^{3,1}P_J$ multiplet, only one with $J = 2$ can be associated with $Z(3930)$, which was observed as a regular resonance, $X(3872)$ looks like a sharp cusp, and two others are not seen in this region. It is important to explain this very different behavior.

On the theoretical side there are models based on the $D_0D_0^*$ molecular picture of $X(3872)$ [8–11] and the tetraquark system [12]; see [13] for a review. However, one cannot get a simultaneous explanation of points (1)–(4) from these models, and we develop here an alternative approach. It is a purpose of this Letter to exploit a realistic dynamical mechanism, constructed in [14], which can explain all four points. Below we shall Briefly explain the mechanism of channel coupling (CC) with the decay channels [14]. This method allows us to calculate not only position of poles in the CC system, like $Q\bar{Q}$ and $Q\bar{q}$, $\bar{Q}q$, but also scattering amplitudes and production cross sections. We demonstrate that, in the 2^3P_1 CC system coupled by the S -wave decays, two poles originating from complex conjugate Breit-Wigner resonances of $Q\bar{Q}$ system are shifted by CC to the final position, with one

pole yielding a narrow cusp at one of thresholds, and another a shifted flat background. We show that when the CC coupling increases, the latter pole yields a very broad bump, and at the same time the weak threshold cusp at the higher threshold $D_+D_-^*$ goes over into a sharp peak at the lower one $D_0D_0^*$. In this way the same value of the coupling constant γ (well within the accuracy limits of the universal constant fitted to different charmonium and bottomonium states in [15]) produces the visible effects, compatible with the properties (1)–(4) mentioned above. To produce this effect as in the 2^3P_1 state, the original single-channel pole should be above and in the attraction region of the threshold. For the poles originally below threshold, CC shifts poles down, as it happens with the 2^3P_2 pole. The same approach allows us to explain the situation with two other poles, 2^3P_0 and 2^1P_1 , as will be discussed below.

Resonances in coupled channels can exist for 3 different reasons [16]: (a) due to bound states in the $Q\bar{Q}$ channel, which are shifted by CC, (b) due to poles in the $(Q\bar{q})(\bar{Q}q)$ channel, shifted by CC; (c) due to strong CC alone (even if no interaction exists in decoupled channel). The most striking feature of the CC resonance is that it approaches the threshold at increasing coupling and typically looks like a pronounced cusp at the threshold of small width. In the realistic physical problem several of these reasons can be present at the same time: e.g., the bare state in the single-channel charmonium can be shifted by strong CC exactly to the threshold. This situation will be discussed below and is characteristic for the single-channel pole above the S -wave threshold, where one meets at the same time with the $Q\bar{Q}$, and the strong CC interaction, which shifts the bare charmonium pole exactly to the threshold position and produces a sharp peak. We shall quantitatively describe this situation in the CC formalism of [14], where the only changeable parameter is the channel coupling constant γ being varied around the standard value.

The basis of the CC theory developed in [14, 15] can be shortly formulated in three relations: a) The effective string decay Lagrangian of the 3P_0 type for the decay

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$$Q\bar{Q} \rightarrow (Q\bar{q})(\bar{Q}q)$$

$$\mathcal{L}_{sd} = \int \bar{\psi}_q M_\omega \psi_q d^4x \quad (1)$$

with M_ω tested in charmonium and bottomonium decays [15], $M_\omega \approx 0.8$ GeV where light quark bispinors are treated in the limit of large m_c mass as solutions of Dirac equations, and this allows us to go over to the reduced (2×2) form of the decay matrix element, $\mathcal{L}_{sd} \rightarrow \gamma \int iv_c \sigma \mathbf{p} v d^4x$, $\gamma = \frac{M_\omega}{\langle m_q + U - V + \varepsilon_0 \rangle} \approx 1.4$ (with realistic averages of scalar $U = \sigma r$ and vector $V = -\frac{3}{4} \frac{\alpha_s}{r}$ potentials). b) The decay matrix element of the state n_1 of heavy quarkonium $Q\bar{Q}$ to the states n_2 and n_3 of heavy-light mesons $Q\bar{q}$, $\bar{Q}q$,

$$J_{n_1 n_2 n_3}(\mathbf{p}) = \frac{\gamma}{\sqrt{N_c}} \int \frac{d^3q}{(2\pi)^3} \bar{y}_{123}^{red}(\mathbf{p}, \mathbf{q}) \Psi_{Q\bar{Q}}^{+(n_1)}(c\mathbf{p} + \mathbf{q}) \times \psi_{Q\bar{q}}^{(n_2)}(\mathbf{q}) \psi_{\bar{Q}q}^{(n_3)}(\mathbf{q}). \quad (2)$$

Here all wave functions ($\Psi_{Q\bar{Q}}$, $\psi_{Q\bar{q}}$, $\psi_{\bar{Q}q}$) refer to the radial parts of the corresponding wave functions, while \bar{y}_{123}^{red} comprises the decay vertex of \mathcal{L}_{sd} and all spin-angular parts of mesons involved; the list of \bar{y}_{123}^{red} for the 6 lowest states is given in Table VII of [14]. In Eq.2 $c = \frac{\omega_Q}{\omega_q + \omega_Q}$, where the averaged kinetic energies of heavy and light quarks in D meson $\omega_q \simeq 0.55$ GeV, $\omega_Q \simeq 1.5$ GeV are taken from [17]. c) The CC interaction “potential” $V_{n_2 n_3}^{CC}$ between $Q\bar{q}$ and $\bar{Q}q$ mesons due to intermediate states of bound $Q\bar{Q}$ system,

$$V_{n_2 n_3}^{CC}(\mathbf{p}, \mathbf{p}', E) = \sum_n \frac{J_{n_2 n_3}^+(\mathbf{p}) J_{n_2 n_3}(\mathbf{p}')}{E - E_n} \quad (3)$$

and the final $(Q\bar{q})(\bar{Q}q)$ Hamiltonian looks like $H = H_0 + V^{CC}$, where H_0 may contain the direct $Q\bar{q}$ and $\bar{Q}q$ interaction, which is $\mathcal{O}(1/N_c)$ and we disregard it in what follows. The equation for the pole position is [14]

$$\det[E - \hat{E} - \hat{w}(E)] = 0, \quad (\hat{E})_{mn} = E_n \delta_{mn} \quad (4)$$

where E_n is the mass of the bare states and $w_{nm}(E)$ is

$$w_{nm}(E) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{n_2, n_3} \frac{J_{n_2 n_3}(\mathbf{p}) J_{mn_2 n_3}^+(\mathbf{p})}{E - E_{n_2 n_3}(\mathbf{p})}, \quad (5)$$

with $E_{n_2 n_3}(\mathbf{p}) = E_{n_2} + E_{n_3}$. We shall be interested below in the pole positions, i.e., solutions of (4), and the most important for comparison with experiment, is the production cross section

$$\sigma_{prod} = \sum_n \Phi_n(E) \frac{\text{Im } w_{nn}(E)}{|E - E_n - w_{nn}(E)|^2}, \quad (6)$$

where $\Phi_n(E)$ is a weakly changing function of E. In Figs.2 and Fig.3 this factor is omitted.

We first apply this method to the case of the 1^{++} state of $Q\bar{Q}$, and we confine ourselves to one state 2^3P_1 in the $Q\bar{Q}$ system so that Eq. (4) reduces to

$$E - E_n = w_{nn}(E) \quad (7)$$

and w_{nn} is given in (5), where $J_{nn_2 n_3}(\mathbf{p})$ should be calculated with the wave functions of D and D^* mesons (we disregard here the possible difference in wave functions of D_0 and D_\pm , and of D_0^* and D_\pm^* , and we also disregard all other states n_2, n_3 beyond DD^*). The wave functions of all states involved have been calculated by Badalian et al. [7], using the relativistic string Hamiltonian [18] derived in the framework of the field correlator method [19]. Here only universal input is used: current quark masses m_q , string tension σ and strong coupling α_s . These realistic w.f. ($\Psi_{Q\bar{Q}}$, $\psi_{Q\bar{q}}$, $\psi_{\bar{Q}q}$) have been fitted by a series of oscillator w.f. and in this way both $J_{nn_2 n_3}(\mathbf{p})$ and $w_{nn}(E)$ were numerically obtained.

To understand the nature of singularities in the energy plane, which produce the cusp at the $D_0 D_0^*$ threshold, we find solutions of (7). We separate from $w(E)$ the square root singularity, while the rest is a slowly varying function, which we approximate by $w(E_{th})$

$$w(E) \cong w(E_{th}) - \frac{i\tilde{M}}{2\pi} k |J(0)|^2 \quad (8)$$

where $E = E_{th} + k^2/2\tilde{M}$ and $\tilde{M} = \frac{M_D M_{D^*}}{M_D + M_{D^*}}$ is the reduced mass. Note that $w(E_{th}) < 0$, and the two pole solutions of Eq.(7) are

$$k_\pm = -\frac{ia}{2} \pm \sqrt{-\frac{a^2}{4} + b}. \quad (9)$$

with $a = \tilde{M}^2 |J(0)|^2 / \pi$ and $b = 2\tilde{M} [w(E_{th}) + (E_n - E_{th})]$. Starting with small coupling, one has two Breit-Wigner poles. With increasing γ the square root vanishes at $\gamma = \bar{\gamma}$ and the two poles collide; for $\gamma > \bar{\gamma}$, both poles move apart along imaginary k axis as shown in Fig.1; and at some $\gamma = \gamma^*$, $\gamma^* = \frac{E_n - E_{th}}{|w(E_{th})|}$, the pole k_+ passes zero, providing the sharp peak at the threshold.

The analysis can be extended to the case of two thresholds $E_{th}^{(1)}$ and $E_{th}^{(2)}$ with the resulting equation

$$\left(k_1^2 - b + ik_1 \frac{a}{2}\right)^2 + (k_1^2 - \Delta) \left(\frac{a}{2}\right)^2 = 0,$$

where $\Delta \equiv 2\tilde{M}(E_{th}^{(2)} - E_{th}^{(1)})$. For small Δ the analysis goes as before, and the only difference in the k_1 plane is the appearance of the cut connecting points $k_1 = \pm\sqrt{\Delta}$, which denote access to the second sheet of k_2 . As before, the trajectory of the highest pole ($k_1^{(+)}$) passes through the origin, leading to a sharp cusp at $E_{th}^{(1)}$. We have found that the pole never passes through the point $E_{th}^{(2)}$, implying that the peak at $E_{th}^{(2)}$ is never so high, as at $E_{th}^{(1)}$, compare curves (2) and (4) in Fig.2b. These curves

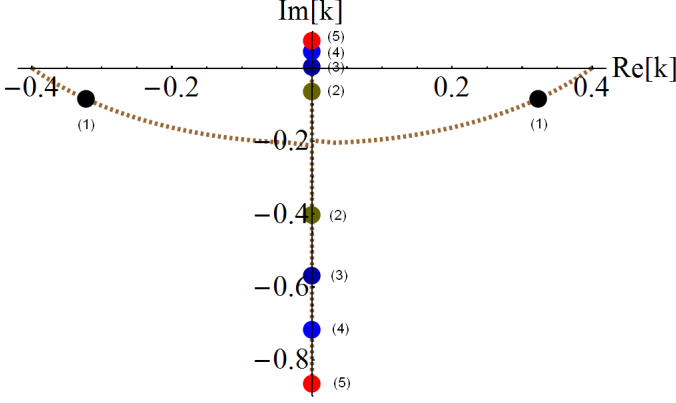
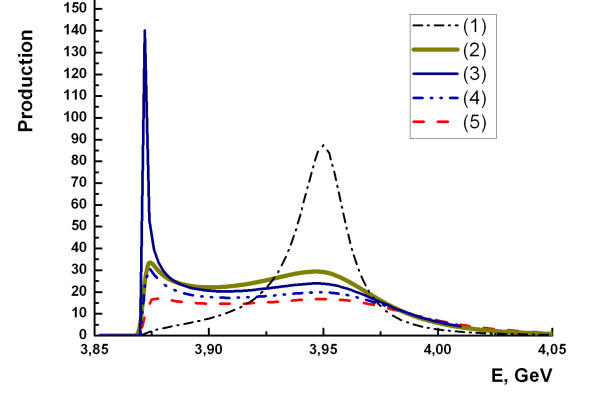


FIG. 1: Motion of poles k_+ (starting on right-hand side) and k_- (starting on left-hand side) in the k plane (in units GeV) with growing coupling parameter γ . Numbers at field circles on curves correspond to numbers on curves in Fig. 2a.

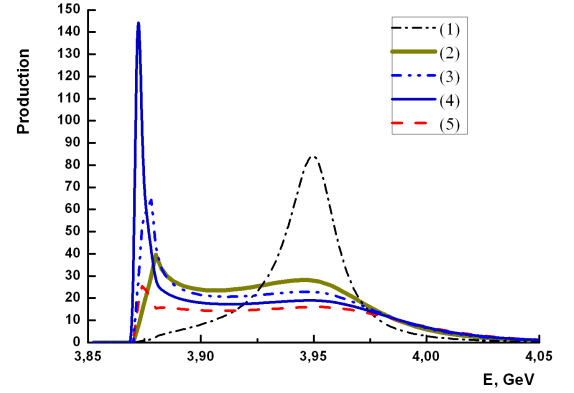
correspond to the situations when the pole is closest to $E_{th}^{(2)}$ and when the pole passes the origin, respectively.

Summarizing this analysis, we have found the pole structure behind the phenomenon of $X(3872)$, and we may assert that the sharp peak at 3872 is due to the pole $k_1^{(+)}$ very close to the $D_0 D_0^*$ threshold, which originally was a genuine Breit-Wigner pole, and the flat background contains the far virtual pole $k_1^{(-)}$, originally the complex conjugated Breit-Wigner pole generated by the same charmonium $2^3 P_1$ state at ~ 3950 MeV, and shifted to the final position $k_1^{(-)}$ by CC. We stress that the necessary condition for the threshold cusp of the type of $X(3872)$ is $|E_n - E_{th}| \approx |w(E_{th})|$, which means that the strength of CC should be as large as the distance of the original $Q\bar{Q}$ bound state from the threshold. In other words, the $Q\bar{Q}$ pole should be “within reach” of CC interaction. One can also see from (9) that for $E_n < E_{th}$ the radicand is negative, and for the growing γ the pole k_+ moves up, farther from threshold, so that for moderate γ both poles are far from threshold.

In the case of two distinct thresholds $w(E)$ contains two isotopically equivalent thresholds, which we take into account with equal weights. To compare with experiment, we have used the $Q\bar{Q}$ production cross section (6), where $Q\bar{Q}$ (in our case $c\bar{c}$) are produced in some primary reaction, e.g. in e^+e^- double charmonium production or from $B \rightarrow K X$ and then the $Q\bar{Q} \rightarrow (Q\bar{q})(\bar{Q}q)$ transition takes place. The resulting form of the cross section is shown in Fig.2b for five different values of γ . One can see from Fig.2(b) that for weak coupling [curve (1)] only the single-channel charmonium state $E_n(2^3 P_1)$ is seen with $\Gamma \sim 35$ MeV, and the next two curves display a cusp at $E_{th}^{(2)}$ and an almost disappeared $Q\bar{Q}$ resonance, while curve (4) clearly signals a strong cusp at $E_{th}^{(1)}$ and no other features. At even stronger CC [curve (5)] the CC pole goes away from thresholds and the whole picture flattens. Thus we see that the experimental situation is



(a) One threshold, $E_{th}(D_0 D_0^*) = 3.872$ GeV.



(b) Two thresholds, $E_{th}(D_0 D_0^*; D_+ D_-^*) = 3.872; 3.879$ GeV.

FIG. 2: Production [see Eq. 6] (in units GeV^{-1}) for 1^{++} state with different values of channel coupling parameter [(1) $\gamma = 0.6$, (2) $\gamma = 1.0$, (3) $\gamma = 1.1$, (4) $\gamma = 1.2$, (5) $\gamma = 1.3$]. For small values of channel coupling parameter γ [curve (1)] one can see a good Breit-Wigner shape, which corresponds to the shifted $2^3 P_1$ state, while for larger γ [curves (3) and (4)] there is a broadening of higher resonance together with steep rise near the threshold $E_{th}(D_0 D_0^*) = 3.872$ GeV.

well reproduced by curve (4). The positions of both poles changing with γ are marked in Fig.1. One can easily see how the pole k_+ produces the sharp cusp in position (3) for one threshold treatment, corresponding to curve (3) in the production cross section in Fig.2(a).

Having found the mechanism, producing the peak at the $D_0 D_0^*$ threshold, one may wonder what happens with other states of the $n = 2$ $^3 P_J$ family, $J = 0, 2$. To this end one should first estimate the position of bare poles E_n (see Table I). We use the results of Badalian et al. [7] with the slightly modified spin-orbit interaction.

We take now the $2^3 P_2$ bare state, which is mostly connected with the $D^* D^*$ channel, while the DD state is in the D wave and can be neglected. One can see, that $E_n(2^3 P_2) < E_{th}(D^* D^*)$, and $w(E)$ in (5) is real and neg-

TABLE I: Hadronic shift (MeV) of charmonium $2^{3,1}P_J$ bare states E_n for different channels. The bare positions were taken from Badalian et al. [7], δ is the total shift and κ is the closed channel (D^*D^*) suppression coefficient.

State	J^{PC}	E_n	Shifts ($\gamma = 1.1$)				E	Exp.
			DD	DD^*	D^*D^*	δ		
2^3P_2	2^{++}	3969	-	-	$\kappa = 0$	0	3.969	
			-	-	$\kappa = 0.25$	-14	3.955	Z(3930)
			-	-	$\kappa = 0.5$	-27	3.942	
2^3P_1	1^{++}	3954	to threshold				3.872	X(3872)
2^3P_0	0^{++}	3918	-2	-	$\kappa = 0$	0	3.916	
			-2	-	$\kappa = 0.25$	-33	3.883	-
			-2	-	$\kappa = 0.5$	-66	3.850	
2^1P_1	1^{+-}	3959	-	-25	$\kappa = 0$	0	3.934	
			-	-29	$\kappa = 0.25$	-7	3.927	-
			-	-32	$\kappa = 0.5$	-14	3.918	

ative. Hence, with increasing coupling the pole is shifted down, away from the threshold. This is shown in Table I. In terms of our previous analysis, using Eq. (9) with $b < 0$, one can see that the pole is on the imaginary axis. Moreover, one can estimate that $\sqrt{|b|} \gg \frac{a}{2}$ and the near-the-threshold approximation (8) is not applicable; one can better use the original Eq. (7), which yields a shift of $|w(E_n)| \sim 55$ MeV. At this point one should take into account the necessity of renormalizing the contributions of higher closed thresholds (which otherwise produce unacceptable shifts, see [14, 20, 22]). Therefore we introduce the coefficient κ , which multiplies in (3) the contribution of the closed channel D^*D^* . We estimate κ in an approximate range $0 \leq \kappa \leq 0.5$ because it gives the most sensible results for the mass shift of J/ψ . However κ may depend on quantum state and bare energy. The resulting position is near the experimentally found [6] Z(3930) peak, as seen in Table I.

A similar situation occurs for the 2^3P_0 state, where the lower threshold is far below, $|E_n(2^3P_0) - E_{th}(DD)| \sim 200$ MeV, while the higher threshold is more distant, than in the case of the 2^3P_2 state. Our calculation for the shift of the 2^3P_0 state yields a large value $\Delta E \sim 68$ MeV for $\kappa = 0.5$, see Table I, so the final position is 3.850 MeV, which possibly corresponds to the position of the wide peak in $e^+e^- \rightarrow J/\psi D\bar{D}$ in [21]. This enhancement of the shift is due to a much larger overlap matrix element in the 3P_0 state. A similar situation was discussed in [22].

An interesting situation occurs in the case of the 1^{+-} state with the 2^1P_1 pole at bare position 3959 MeV. Here the coupling to the DD^* channel is much weaker, than in the case of the 1^{++} state, and the main coupling is with the closed D^*D^* channel, which defines the destiny of the bare pole. When we renormalize the contribution of the D^*D^* channel with the coefficient κ , as discussed above, the final shift for $\kappa = 0.5$ is around 45 MeV. In Fig. 3 we demonstrate how the production cross section

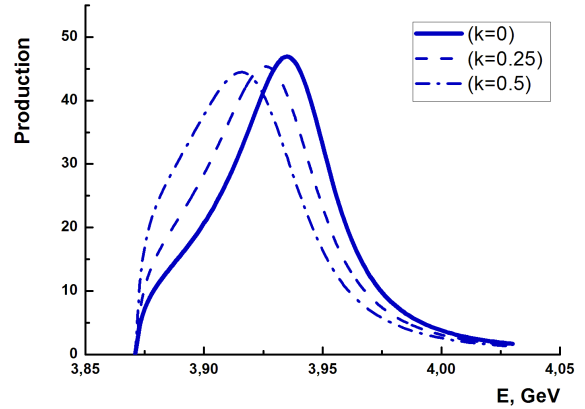


FIG. 3: Production (see Eq. 6) [in units GeV^{-1}] for the 1^{+-} state in DD^* channel for channel coupling parameter $\gamma = 1.1$ and with different values of closed D^*D^* channel suppression coefficient, $\kappa = 0; 0.25; 0.5$.

changes with κ , and one can see, that the resulting width at $\kappa = 0.25 \div 0.5$ is around $\Gamma \sim 50$ MeV.

In conclusion, we have calculated the amplitudes of CC processes connecting $Q\bar{Q}$ and $(Q\bar{q})(\bar{Q}q)$ systems via the decay matrix element, Eq.(2), involving realistic wave functions of all hadrons involved, and the CC constant $\gamma(M_\omega)$, fitted earlier to bottomonium and charmonium transitions. For the concrete case of the 1^{++} state of charmonium we have found pole structure and production cross section. At small CC two poles correspond to the complex conjugated poles of one Breit-Wigner resonance of the 2^3P_1 state of $Q\bar{Q}$ with the width $\Gamma \sim 35$ MeV. For increasing CC one of the poles approaches the thresholds and another moves away, as a result this bare resonance flattens, while a sharp cusp appears first at the $D_+D_-^*$ and then at the $D_0D_0^*$ threshold at ~ 3872 MeV. This latter situation with the sharp narrow cusp at the $D_0D_0^*$ threshold and absence of any other structures (except for a tiny cusp at higher threshold) including the region around 3940 MeV corresponds to the observed production yield [6]. We conclude that our dynamical mechanism explains properties (1)-(4), in particular, why the resonance X(3872) is at the lower, but not the higher threshold, why it is so narrow, and why the original 2^3P_1 state of charmonium is not seen in experiment.

An alternative and close in spirit approach was developed recently in [22, 23]. Our analysis partly supports the conclusion in [24], that “X(3872) may be of ordinary $c\bar{c}$ 2^3P_1 state origin”. Our results differ from those of [25], where two 1^{++} states were found, one associated with X(3872), and another with X(3940). In a recent review [26] the CC analysis of the X(3872) and X(3940) was reported with the conclusion, that both states cannot be reproduced in the exploited model simultaneously. This result is in common with ours, since in our case the broad enhancement due to the second pole is near the

$D_0 D_0^*$ threshold and cannot be associated with $X(3940)$.

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